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Review

Generalized moist potential vorticity and its application in the analysis of atmospheric flows

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Abstract

Potential vorticity (PV) serves as an important dynamic tracer for large-scale motions in the atmosphere and oceans. Significant progress has been made on the understanding and application of PV since the work of Hoskins et al., who introduced an "IPV thinking" of a dynamical system in a purely dry atmosphere. In particular, there has been a substantial amount of work done on the PV in a general moist atmosphere. In this paper, the generalized moist potential vorticity (GMPV) and its application in the mesoscale meteorological fields are reviewed. The GMPV is derived for a real atmosphere (neither completely dry nor saturated) by introducing a generalized potential temperature instead of the potential temperature or equivalent potential temperature. Such a generalization can depict the moist effect on PV anomaly in the non-uniformly saturated atmosphere. The effect of mass forcing induced by rainfall on the anomaly of GMPV is also reviewed and a new dynamic variable, the convective vorticity vector (CVV), is introduced in connection with GMPV.

Keywords: Potential vorticity dynamics; Mesoscale meteorological dynamics; Non-uniformly saturated atmosphere; Generalized moist potential vorticity; Convective vorticity vector

1. Introduction and general background of potential vorticity (PV) dynamics

The concept of PV can be traced back to the research work of pioneers like Kelvin, Bjerknes, Kibel, Rossby, Ertel, Charney, and Kleinschmidt. The simplest version of the modern concept of PV was given by Rossby [1],

$$P = \frac{\zeta_a}{h}$$

where h is the depth of a material fluid column, $\zeta_a = \zeta + f$ is the vertical component of the absolute vorticity. Rossby's result was given full hydrodynamical generality by the independent work of Ertel [2]. Ertel's derivation led to a general form of PV,

$$P = \rho^{-1}\zeta_a \cdot \nabla\theta$$

where ρ and θ are, respectively, the density and potential temperature of dry air, and ζ_a is a three-dimensional absolute vorticity vector. Another historical landmark was the introduction of isentropic potential vorticity (IPV) concept. The process of mapping out the upper air PV on isentropic surfaces was initiated in the important papers by Reed and Sanders and Reed [3,4]. The computer-generated IPV maps were first given by Obukhov [5] and Danielsen [6,7] for the 300, 305, and 310 K isentropic surfaces. The usefulness of "IPV thinking" was discussed in Hoskins et al. [8], in which they showed that this concept stemmed first from the invertibility of the PV field. In Hoskins et al. [8], they suggested that PV not only provides another dynamical field describing a balanced motion, but also is simply advected like a materially conserved chemical tracer in the absence of diabatic heating and frictional force. As a result, three significant properties of PV can be deduced: the conserva-

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tional principle of PV, the invertibility principle, and the impermeability principle. In subsequent years, PV has been increasingly used in the diagnosis of observed atmospheric fields for the understanding of synoptic and large-scale atmospheric motions, atmospheric numerical simulation results, and large-scale oceanic circulations. Nevertheless, PV is not conserved when latent heat release is taken into account in a moist atmosphere. Bennetts and Hoskins [9] first generalized PV into a moist potential vorticity (MPV) defined as,

$$P = \rho^{-1}\zeta_a \cdot \nabla\theta_e$$

by replacing θ with the equivalent potential temperature θ_e. Schubert et al. [10] proved an annihilation of the solenoidal term in the MPV equation, thus led to a conservation of MPV in moist adiabatic and frictionless processes. The MPV concept has extensively been used in the studies of conditional symmetric instability [11–14], and in the generation of MPV in extratropical cyclones [15].

In fact, in a mesoscale convective system, air is neither dry nor saturated. The real atmosphere is typically non-uniformly saturated. For this situation, Gao et al. [16] extended the MPV into a generalized moist potential vorticity (GMPV) by replacing potential temperature θ with a generalized potential temperature θ^* (will be defined in Section 4), which has been applied in the $\boldsymbol{Q}$ vector and in the Richardson formula to study strong convection and instability in the non-uniformly saturated atmosphere [17,18].

2. The properties of PV

Since θ is a material surface, the gradient of θ represents a layer of mass trapped on the two material surfaces. It can be seen from its various formulations that PV always captures the rotation part of a layered air, regardless of it being dry or moist. Because of the stable stratification in a normal atmosphere, $\nabla\theta$ is usually directed nearly in the vertical axis. Therefore, PV is a measure of the component of absolute spin about the vertical axis, including the vertical component of the Earth's rotation. More precisely, PV can be regarded as a measure of the intrinsic "cyclonicity" of an air parcel, re1evant to a stratification-constrained and layered-two-dimensional motion. This is re1ated to the fact that, on each isentropic surface of the stable stratification, PV is proportional to the component of absolute vorticity precisely normal to that surface. Also, because this component of vorticity tends to increase or "spin up" when $|\nabla\theta|$ is decreased by adiabatic vertical motion, and vice versa, the PV value of the air parcel tends to remain constant. Extratropical stratospheric air has a very high intrinsic cyclonicity in comparison with tropospheric air. Therefore, in extratropical explosive cyclogenesis, one of the important causes is extratropical 1ower-stratospheric air descending along a sloping isentropic surface and interacting dynamically with warm, moist lower-tropospheric air [8,19,20]. This phenomenon is known as the high-PV air intrusion or tropopause folding.

The second fundamental point about PV is the idea of its "invertibility". More precisely, there is an "invertibility principle" to the effect that if (a) a suitable balance condition is imposed to eliminate gravity and inertio-gravity waves from consideration, and if (b) a suitable reference state is specified, then a knowledge of the distribution of PV on each isentropic surface and of θ at the lower boundary, is sufficient to deduce, diagnostically, all the other dynamical fields such as winds, temperatures, pressures, and the altitudes of the isentropic surfaces [8].

It should be emphasized here that a local knowledge of PV does not imply a local knowledge of the wind information, expressed either in ψ (streamfunction) or in v (wind), because the inversion is a global process. In particular, it depends on specifying suitable boundary conditions to make the inverse Laplacian ∇^{-2} unambiguous. Also, in this system the balance condition, on which invertibility depends, corresponds simply to the absence of sound or external gravity waves. They have been filtered out by the assumption of incompressible and non-divergent motion. Furthermore, there is a scale effect, whereby small-scale features in the PV field have a relatively weak effect on the ψ and v fields, while large-scale features have a relatively strong effect. In particular, ψ and v are to varying degrees insensitive to fine-grain structure in the PV field [8].

In fact, Ertel's theorem is a particular case of the general result expressed as follows [2,21]:

$$\frac{\mathrm{D}P}{\mathrm{D}t} = -\rho^{-1}\nabla \cdot N_Q \tag{1}$$

where the three-dimensional material derivative and the non-advective flux or transport are defined, respectively, by $\mathrm{D}/\mathrm{D}t = \partial/\partial t + v\cdot\nabla$ and $N_Q = -H\zeta_a - \boldsymbol{F} \times \nabla\theta$ (H is the diabatic heating rate, and $\boldsymbol{F}$ is the viscous or any other non-conservative force).

The flux form of (1), from which (1) itself can be recovered using the mass-conservation equation $\partial\rho/\partial t + \nabla\cdot(\rho u) = 0$, expresses exact conservation and is given by

$$\frac{\partial(\rho P)}{\partial t} + \nabla \cdot \boldsymbol{J} = 0 \tag{2}$$

where $\boldsymbol{J} = v\rho P + \boldsymbol{N}_P$ denotes a total PV flux. The flux form (2) expresses conservation in the most general possible sense. For instance, it is different according to whether or not the system is mass conserving [22].

A pursuit of the PV-chemical transport analogy to its logical conclusion leads to an expression for the flux $\boldsymbol{J}$, showing that PV behaves as if the quasi-molecules particles of signed "PV-substance" (PVS) can be transported along isentropic surfaces, but not across them, and be created or destroyed (apart from "pair production" and "mutual annihilation") only where isentropic surfaces meet at the boundaries. These properties are the counterparts for PV of the total vorticity transport tensor [22–24]. Being completely general, they apply to the real atmosphere in all its complexity.

In summary, PV is conceptually very succinct and involves replacing the concepts of "force" and "torque" by the concept of "PV flux" or "PV transport"—or "generalized rearrangement" of PVS. The phrase "generalized rearrangement" is meant to suggest the horizontal migration of PVS particles confined to each θ-layer, allowing for dilution and concentration effects as mass enters and leaves the layer, and pair production and mutual annihilation, even in the tropics [25].

3. Application of PV in tropical cyclones (TC)

The appealing properties of PV have been extensively applied to the research of TC, especially of the TC motion. TC itself can be regarded as a little "rotating Earth", on which a complete set of Eigen-modes of a dynamic system exists. Montgomery and Lu [26] discussed vortex Rossby waves and gravity waves using a basic state retrieved from a PV inversion. The corresponding PV dynamic system describing TC can be derived from a set of gradient balance approximations [27,28]. Kasahara and Platzmann [29] first pointed out that the presence of a vertical wind shear would result in a horizontal gradient of PV under adiabatic conditions, which can have an effect on TC motion. Based on the IPV thinking, Hoskins et al. [8] illustrated that the ridges and troughs of Rossby wave patterns tend to move to the regions with minimum and maximum PV tendencies, respectively, if the wave intensity does not change during the displacement. Similarly, TC motion can be directly related to the PV tendency. Also, latent heat release causes changes in the vertical PV distribution, which then modifies the vorticity distribution. As a result, the location of maximum relative vorticity tendency, and hence TC motion, would be altered. Monlinari [30] showed that the intensity change of Hurricane Helen could be associated with the evolution of an upper-tropospheric PV anomaly. Reilly [31] conducted an observational case study and found that upper-tropospheric PV advection plays an important role in tropical cyclogenesis. Montgomery and Farrell [32] also investigated the influence of upper-level potential vorticity disturbances on tropical cyclone formation within the context of two simple non-linear balance models that incorporate moist processes. Wu and Emanuel [33] suggested that hurricane might be considered as local sources of nearly zero PV air in the upper troposphere. In their work, PV diagnostics were applied to evaluate the control by the large-scale environment of hurricane movement and, more importantly, to assess the storm's influence on its own track. Wu and Emanuel [34] suggested that conservation of PV would cause a downward/upward penetration of the upper-level anticyclone (low-level cyclone), which creates an additional steering flow. As a result, the TC movement differs from that specified by the environmental flow. A few observational studies on TC motion related to non-barotropic processes have been made and all were based on case studies [35–37]. The general conclusion from these studies is that PV at the upper levels appears to be an important contributor, apparently through the downward penetration of high-PV air.

4. The application of PV in mesoscale convective systems and rainstorms

The application of PV dynamics to mesoscale convective systems (MCS) can be very subtle, because mesoscale motions can retain both balanced and imbalanced flows [38]. Gray [39] discussed mass forcing in generating PV anomalies in MCSs. Nevertheless, several studies have shown that using PV and its associated invertibility principle do capture the main features of the mesoscale convective systems [38,40–42].

Apart from PV downscalability issue, moist convection poses another difficulty for the application of PV dynamics in mesoscale convective systems. In order to solve this problem, a moist potential vorticity (MPV) was thus introduced by replacing potential temperature θ with the equivalent potential temperature θ_e because PV cannot be used to describe the high humidity field. It was shown that motion in a saturated atmosphere without diabatic heating and viscous dissipation conserves moist potential vorticity [10]. This property was then used to investigate the development of vertical vorticity in moist baroclinic processes. From then on, many studies of rainstorms were related with MPV. From these studies, dynamical and diagnostic methods reveal that MPV anomaly regions correspond well to the regions of intensive precipitation.

In the rainstorm, the real situation is that air is typically non-uniformly saturated. In other words, relative humidity does not approach to 100% in the clouds. For such cases, Gao et al. [16] introduced a powerful and useful moist variable as given below. For a non-uniformly saturated atmosphere, the generalized potential temperature is defined as,

$$\theta^* = \theta \exp\left[\frac{Lq_s}{C_pT}\left(\frac{q}{q_s}\right)^k\right] \quad (3)$$

where it is evident that when air is saturated, $q = q_s$, $\theta^* = \theta_e = \theta\exp(Lq_s/C_pT)$; when air is completely dry, $q = 0$, $\theta^* = \theta$; and in the most general case, θ^* will take a value between θ *and* θ_e (depending on the ratio of q/q_s).

Furthermore, the generalized MPV equation can be derived based on the θ^*, and be expressed as,

$$\frac{dP_m}{dt} = \alpha(\nabla p \times \nabla\alpha)\cdot\nabla\theta^* \quad (4)$$

where $P_m = \alpha\zeta_a\cdot\nabla\theta^*$ is the generalized MPV (GMPV). It has been shown that θ^* and GMPV are very powerful thermodynamic and dynamic parameters with great application potential in the study of non-uniformly saturated phenomena, such as in rainstorm, fog, and even heat wave events.

A rainstorm is highly related to a deep convective system with a nearly vertical distribution of moist isentropic surface. Compared with quasi-horizontal distribution of the isentropic surface in the large-scale systems, the two isentropic surfaces in these two atmospheric situations are almost orthogonal. In the latter situation, PV should be modified as $\alpha\zeta_a \times \nabla\theta_e$, i.e., the so-called convective vorticity vector (CVV) [16]. This vector is highly correlated with hydrometers in the deep convective cloud. Gao et al. [43,44] diagnosed the development of tropical convective systems based on 2D or 3D cloud resolving model simulation data. The results seemed very promising.

5. Discussions and conclusions

PV is at the core of balanced atmospheric dynamics. It is conservative in the adiabatic and frictionless conditions; it is easy to express its behaviors by analysis of its advection behaviors. More importantly, in a balanced system, PV has invertibility, and so it can be used to derive the wind field as well as all other thermodynamic fields. All these have brought great convenience to the study of large-scale dynamics problems.

Apart from the large-scale systems that give background fields for weather situation, the real causes leading to disasters or severe weather phenomena are mostly in the sub-synoptic scales, especially mesoscale convective systems, which often result in heavy rains and extreme weather situations. The question now is whether or not it is suitable to make a use of PV to describe or characterize the vigorously developing mesoscale convective systems. If the PV is still a suitable quantity, should it be generalized on the basis of the classic definition of PV? This is a new problem and a new challenge. In order to probe into the above questions, the properties of PV and its applications and related limitations have been analyzed. In this review paper, we have revisited several studies attempting to generalize the classic PV into mesoscale convective systems with moist and even non-uniform moist atmosphere.

It should be noted that in the derivation of PV, when using potential temperature gradient to dot the vorticity equation, the scalar product of the potential temperature gradient and the solenoid term in the vorticity equation automatically becomes zero because potential temperature is a function of pressure and temperature. Although the solenoid production of vorticity (so-called the internal vorticity source) originally exists in the vorticity equation, because of a scalar product, it becomes zero in the PV equation. The merit in doing so is to make the PV equation a conservative under adiabatic and frictionless conditions, but the internal source of the vorticity generation is lost during this process. The process of using the potential temperature gradient to dot the vorticity equation, and eventually becoming a PV equation, is also a process of changing the vector equations of vorticity into a scalar equation. Obviously, compared with a vector (e.g., the vorticity), a scalar (e.g., the PV) has its own weakness; a scalar quantity has only magnitude, while a vector has magnitude and direction. It is a delicate issue for mesoscale convective systems that whether it is important to maintain the internal source terms of vorticity or to have a scalar PV conservation law. Therefore, the concept of GMPV and CVV may share some important insight into these questions.

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